**What Is Dynamic Hedging?**

Hedging involves mitigating the variability in returns of an uncovered position by entering related investment positions. For example, stock prices fluctuate, options depend on the underlying asset’s price, and fixed income positions face risks like credit or interest rate exposure. A "perfect hedge" eliminates risk entirely, such as an oil producer selling a forward contract to lock in a future price. However, perfect hedges are rarely feasible with complex financial instruments.

Static hedging, like the oil producer example, involves setting a hedge that requires no further management. Dynamic hedging, however, requires continual rebalancing to maintain a desired risk profile, such as risk neutrality. For stock options, this rebalancing often involves trading in the underlying asset. This approach is guided by the Black-Scholes model, which provides an analytical framework for option pricing and defines how the value of an option depends on various underlying factors.

The Black-Scholes formula for a European call and put option is as follows:

*For a call option:* *For a put option:*

*Where:*

Here, is the current stock price, is the strike price, is the time to maturity, is the risk-free rate, is the volatility of the stock, and is the cumulative distribution function of the standard normal distribution. The Greeks, derived as partial derivatives of the Black-Scholes formula, measure the sensitivities of an option’s value to these inputs. Delta, which measures the sensitivity of an option’s price to changes in the stock price , is particularly important for dynamic hedging.

Dynamic delta hedging involves maintaining a portfolio with a negative Delta equivalent to the option’s underlying stock position to neutralize risk from price changes. However, Delta is not constant; it evolves over time and is influenced by factors like implied volatility, time to maturity, and strike price. For example, as options approach maturity, Delta becomes more sensitive to price changes, especially for at-the-money (ATM) options. In contrast, deep in-the-money (ITM) or out-of-the-money (OTM) options exhibit less sensitivity.

For call options, Delta ranges from 0 (OTM) to 1 (ITM), with ATM positions having a Delta near 0.5. For put options, Delta ranges from 0 to -1, following a similar pattern. Implied volatility impacts Delta significantly: higher volatility raises the probability of options finishing ITM, thereby increasing Delta. Time to maturity also plays a critical role; shorter-dated options exhibit greater Delta sensitivity, while longer-dated options are less responsive due to the extended time horizon.

The Black-Scholes model underpins these relationships, providing the foundation for understanding how changes in variables like volatility, time, and underlying price affect option pricing and Delta. The goal of dynamic delta hedging is to maintain a delta-neutral portfolio, minimizing exposure to directional price movements while capturing profits driven by volatility. By continuously adjusting the hedge as Delta evolves, traders can mitigate risk and adapt to market conditions. Further considerations for implementing this strategy will be explored in later sections.

**How do People Treat Delta Hedging?**

Delta hedging is widely regarded as a versatile risk management tool, with applications that have evolved significantly since the introduction of the Black-Scholes model in 1973. Originally designed to neutralize price sensitivity for individual options, it now extends to hedging entire portfolios. By aggregating the deltas of all positions, portfolio managers can assess overall market sensitivity and construct hedges with options or futures to offset directional risks. The dynamic nature of delta hedging enables continuous adjustments to adapt to changing market conditions, stabilizing portfolio values. However, frequent rebalancing incurs costs and leaves residual exposure to higher-order risks like gamma and vega.

In statistical arbitrage, delta hedging is leveraged to exploit market inefficiencies by isolating pricing anomalies while maintaining market neutrality. By replicating portfolios through delta hedging, traders can capitalize on price mismatches between stocks and options, or other discrepancies. Advanced computational tools and high-frequency trading strategies facilitate rapid adjustments, allowing practitioners to capture fleeting opportunities. Unlike portfolio management, where delta hedging prioritizes risk reduction, its use in statistical arbitrage is profit-driven, accepting higher rebalancing costs to maximize returns.

**The Limitations of Non-ML Dynamic Hedging Approaches**

The Black-Scholes Model (BSM) faces several limitations when applied to dynamic hedging, stemming from both its analytical framework and the practical realities of implementation. One significant issue is the analytical flaw of time-discretization. Hedge ratios derived from the Black-Scholes equation rely on first-order Taylor approximations of stock price changes. While this approximation becomes more accurate as the time between rebalancing decreases, continuous rebalancing is impossible in practice. Consequently, the effectiveness of these hedge ratios diminishes when large stock price movements occur between rebalancing periods.

In practice, the operational costs of time-discretization further complicate dynamic hedging. Frequent rebalancing incurs substantial transaction costs, including bid-ask spreads and fees, making continuous hedging infeasible. Investors are incentivized to rebalance less frequently, but this exacerbates the approximation errors introduced by time-discretization. These constraints highlight the trade-off between reducing risk through frequent adjustments and managing transaction costs.

BSM also assumes constant volatility, which is unrealistic in real-world markets. Practitioners often adopt techniques like delta-vega hedging, which involves trading multiple options to offset sensitivity to both Delta and Vega. This strategy, while theoretically sound, is operationally constrained by low liquidity and slippage, which can significantly increase transaction costs. More advanced hedging strategies, such as delta-gamma or delta-vega-gamma hedging, extend this approach to address higher-order risks. However, these methods remain expensive and highly dependent on market conditions.

Another critical limitation of BSM is its assumption of lognormal stock price behavior. In reality, stock prices often exhibit "fat tails," with more frequent extreme events than a lognormal distribution would suggest. This mismatch can lead to significant errors in pricing and hedging. Additionally, the volatility smile observed in markets contradicts BSM’s assumption that strike price and time to maturity have no effect on volatility. Implied volatility tends to decrease as strike prices increase initially, before rising again, creating the characteristic smile shape. To address this, models such as implied binomial trees, deterministic volatility, and path-dependent volatility have been developed. While these models improve accuracy, they often result in larger replication errors for delta-neutral portfolios due to persistent pricing biases in BSM.

Ad hoc models have also been introduced to refine BSM’s assumptions. Techniques like nonlinear least squares (NLS) minimize errors between market and model prices, while models such as constant elasticity of variance, jump-diffusion, and stochastic volatility add complexity to account for real-world behaviors. However, despite their improvements, these models often fail to fully capture market dynamics or changing asset behavior over time.

Ultimately, the limitations of BSM in dynamic hedging are rooted in its simplifying assumptions and the operational constraints of real-world implementation. As markets evolve and become increasingly complex, machine learning offers a promising alternative for developing dynamic hedging strategies that can better adapt to the intricacies of financial markets. These methods will be explored in the following section.

**The Evolution of Dynamic Hedging: How Machine Learning Has Surpassed Limitations**

Dynamic hedging has evolved significantly since its inception alongside the Black-Scholes model in 1973. Initially, this model provided the theoretical foundation for option pricing, enabling widespread adoption of financial derivatives and hedging strategies aimed at achieving risk neutrality. During the 1980s, derivatives gained prominence, with static hedging strategies still relying heavily on the Black-Scholes framework. These methods remained largely unchanged until the late 1990s, when the field of quantitative finance introduced mathematical solutions such as lattice methods, Monte Carlo simulations, and finite difference techniques (Broadie & Glasserman, 1997). These approaches marked the advent of dynamic hedging by allowing strategies to adapt to market conditions. However, they were limited by their reliance on rigid assumptions, human discretionary inputs, and operational inflexibility, which hindered their performance in real-world scenarios.

The early 2000s brought the emergence of high-frequency trading (HFT), revolutionizing dynamic hedging strategies. HFT allowed for instantaneous and automated adjustments to hedging positions, reducing human error and enabling traders to account for nuances like market microstructure and slippage (Gerig, SEC paper). By operating at smaller time intervals, HFT enhanced responsiveness to market conditions. However, this approach introduced new risks, as demonstrated by events like the 2010 Flash Crash, where algorithmic trading caused extreme mispricing. Despite its advancements, the limitations of HFT highlighted the need for more robust methods.

The introduction of machine learning in the early 2010s marked a pivotal shift in dynamic hedging. Support Vector Regression (SVR) enabled modeling of non-linear relationships, improving forecasting accuracy for implied volatility and delta adjustments (Basak et al., 2011). Decision trees and random forests brought interpretability and robustness, allowing traders to segment data into volatility regimes and reduce overfitting in noisy environments (Breiman, 1984; Bali et al., 2009). These innovations enhanced the predictive power of dynamic delta hedging strategies, particularly in volatile markets, and laid the foundation for deeper learning approaches.

By 2015, neural networks emerged as transformative tools in dynamic hedging. Advances in computational power, data availability, and training techniques enabled architectures like feedforward networks, convolutional neural networks (CNNs), and recurrent neural networks (RNNs) to model non-linear relationships and temporal dependencies in market data (LeCun et al., 2015). These methods excelled at managing path-dependent options, such as Asian and barrier options, by leveraging features like hierarchical data extraction and improved robustness through techniques like dropout (Hinton et al., 2012) and batch normalization (Ioffe & Szegedy, 2015). Neural networks significantly enhanced the accuracy of delta prediction, facilitating more efficient recalibration of hedging positions. However, their inability to directly incorporate transaction costs remained a limitation, later addressed by reinforcement learning (RL).

Reinforcement learning has become a cornerstone of modern dynamic hedging, with deep reinforcement learning (DRL) methods revitalizing its application. DRL integrates the decision-making capabilities of traditional RL with the feature extraction power of deep learning, enabling the development of adaptive, data-driven hedging strategies (Mnih et al., 2015). Models like Deep Q-Networks (DQN) and Policy Gradient Methods explicitly account for transaction costs and liquidity constraints, balancing risk and reward (Kolm & Ritter, 2020). DRL frameworks also leverage simulated environments to train on historical and synthetic data, ensuring robustness across diverse market conditions, including stress events. This capability mitigates the risks associated with relying solely on historical data, which may not fully represent future market behavior.

A graph of a graph showing different colored circles

Description automatically generated with medium confidence

While neural networks and RL share transformative potential, their strengths and limitations differ. Neural networks excel at capturing complex, high-dimensional relationships and are particularly effective for predicting option delta and managing non-linear dependencies. However, they require frequent retraining to remain effective in dynamic environments. RL, in contrast, is inherently adaptive, refining optimal hedge ratios in response to evolving market conditions. Its ability to explicitly incorporate transaction costs makes it well-suited for real-world scenarios. Despite their computational intensity and reliance on robust training environments, RL methods offer a more holistic approach to addressing the complexities of dynamic hedging in modern markets.

The evolution from static hedging to machine learning-driven strategies highlights the field's trajectory toward more adaptive and robust approaches. By integrating advanced computational techniques, practitioners can better navigate the intricacies of dynamic hedging, overcoming many of the limitations of traditional models while aligning strategies with real-world constraints.

**Choice of ML for our Case Study: Neural Networks**

Reinforcement Learning and Neural Networks are two prominent machine learning approaches for dynamic hedging. For this case study, our group selected Neural Networks for several reasons.

First, we prefer ML applications that rely on universal data, such as options data, rather than firm-specific details like transaction costs, liquidity constraints, or capital requirements. Data on highly liquid options (e.g., SPX options) is readily available, whereas firm-specific data is often inaccessible and would need to be fabricated. By excluding firm-specific features, our model avoids niche applicability and is more generalizable to a variety of trading environments. While Reinforcement Learning is particularly adept at integrating firm-specific operational constraints into its decision-making processes through penalty terms, this benefit is irrelevant to our chosen scope.

Second, Neural Networks align better with the academic context of this case study, as they can be tied to traditional financial analyses like the Black-Scholes model. Neural Networks are well-suited for feature extraction and inference, enabling insights into key Black-Scholes variables (e.g., volatility and risk-free rate) and outputs such as Delta and Vega. This interpretability allows Neural Networks to provide direct commentary on established analytical frameworks. Reinforcement Learning, by contrast, focuses on generating decisions across a pre-defined feature space, offering less insight into individual features. In many cases, Reinforcement Learning models rely on Neural Networks to define their feature space, underscoring Neural Networks’ foundational importance in dynamic hedging.

Lastly, Neural Networks are included in the curriculum of this Machine Learning course, whereas Reinforcement Learning is not. While this is a pragmatic consideration, it adds further justification for selecting Neural Networks in this project.

Given these factors, our group chose Neural Networks for their ability to model optimal hedge ratios effectively, provide insights into Black-Scholes features, and remain relevant to a broad range of trading scenarios.

**Specific Neural Network Application:**

Drawing inspiration from “*Enhancing Black-Scholes Delta Hedging via Deep Learning*” by Chunhui Qiao and Xiangwei Wan, we use a neural network to improve the Black-Scholes Delta Hedging strategy. Our data, described in detail later, includes features from various SPX call options. For each option, the optimal delta hedge ratio for any given time period is calculated as:

where is the call price, is the underlying asset price, and is the estimated continuous dividend yield.

Directly predicting this optimal hedge ratio might seem intuitive, but as options near maturity, the ratio becomes highly discontinuous around the strike price. To address this, we instead use the residual between the Black-Scholes Delta Hedging Ratio and the Optimal Delta Hedging Ratio as the target variable:

This residual provides a smoother target for the neural network, with observations input cross-sectionally to capture deviations effectively.

**Hands-On**

**Future Extension**

To extend this machine learning application beyond the scope of our current demonstration, several enhancements are necessary.

First, the feature set should be expanded to incorporate additional information. While our dataset captures several features, the paper by Chunhui Qiao and Xiangwei Wan includes other Black-Scholes variables, such as Vega, Gamma, and Theta, which could provide deeper insights for the model.

Second, the scope of the dataset must grow significantly. A robust approach would include options across a wider range of strike prices and, more critically, span years rather than months. This extended dataset would enable the model to learn patterns across varying market conditions. To put this into perspective, our dataset comprises 1,008 unique observations, while the Qiao and Wan study uses 2.073 million.

Lastly, the loss function should directly target the hedging problem. Currently, we minimize the Mean Absolute Error (MAE) of the residual, but the ideal approach would minimize the actual hedging error, defined for each observation as:

This custom loss function would better align the neural network’s optimization with the practical goal of reducing hedging errors.

Implementing these extensions—expanding features, scaling the dataset, and refining the loss function—would enhance the robustness and effectiveness of this application, making it better suited for real-world dynamic hedging challenges.

**Conclusion**

**Member Contributions**